KEY - Exponentials Review

Alg 2C

A. Growth or Decay

1) Growth
since $b = 3$ which is bigger than 1

2) Decay
since $b = \frac{1}{3}$ which is smaller than 1

3) Growth
since $b = 2$ which is bigger than 1

4) Decay
since $b = 0.7$ which is smaller than 1

5) Growth
since $b = 1.2$ which is bigger than 1

6) Decay
since the y-values in the table are getting smaller

7) Growth since the y-values are getting bigger as I read the graph from left to right

B. % Growth + Decay - Word Problems

1) A. Growth because the problem says the money appreciates

B. $y = 250(1.05)^x$

$a = \$250$ since that's the amount of $ I start with

$b = 100\% + 5\% = 105\% \Rightarrow 1.05$ as a decimal

Plug this a and b into $y = a \cdot b^x$

C. $\approx \$335.02$

plug in $x = 6$

$y = 250(1.05)^6 = 335.02$

↑ plug into calculator

2) 423 rabbits

First, write the equation

$a = 500$ rabbits at the start

$b = 100\% - 8\% = 92\% \Rightarrow 0.92$ as a decimal

subtract since they're dying → decay

So $y = 500(0.92)^x$

Then, plug in 2 for x for the 2 years

$y = 500(0.92)^2 = 423.2 \Rightarrow \approx 423$ rabbits

3) $M(y) = 8,500(0.86)^{\frac{y}{2}}$

$a = 8,500$ since that's the amount of $ I start with

$b = 100\% - 14\% = 86\% \Rightarrow 0.86$ as a decimal

∴ subtract because "depreciates" means decay

- I need to make the exponent $\frac{y}{2}$ because y represents each year, but the mutual fund only depreciates every 2 years.

C. Exponent Rules

1) f^{11} - Add the exponents $f^8 f^2 f^1$ $8+2+1=11$

2) $3y^3$ - Subtract the exponents $\frac{3y \cdot y \cdot y \cdot y \cdot y}{y \cdot y} = 3y^3$

3) $25h^8$ - Distribute the second power to each piece.
Multiply the exponents for power to a power

or $(5h^4)^2 = (5h^4)(5h^4) = 5 \cdot 5 \cdot h^4 \cdot h^4 = 25h^8$

4) $\frac{1}{r^5}$ For negative exponents, move the pile across the fraction bar and change the sign on the exponents.

5) 1 Anything to the zero power equals 1

6) $3x^2 + 5x$ - Already simplified. There are no like terms.

7) $3x^3$ - Combine like terms. (Don't change the exponent!)

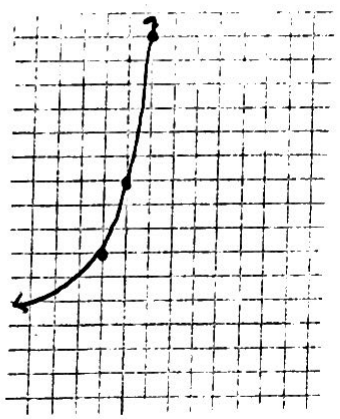
8) $\frac{25^x}{5^{3y}} = \frac{(5^2)^x}{5^{3y}} = \frac{5^{2x}}{5^{3y}} = 5^{2x-3y} \overset{\text{given}}{=} 5^3$ (125)

D - Graphing Exponential Functions

- For these, create a table by hand or use 2nd Graph to get to the table on your graphing calculator.

1)

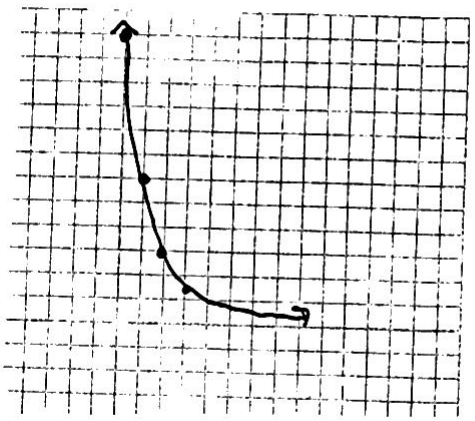
x	y	
-1	3	
0	6	×2
1	12	×2
2	24	



2)

x	y	
-1	24	
0	12	×½
1	6	×½
2	3	

a



6. Solving Exponential Equations

1) $x = 0$

$4^{7x+2} = 16$

$4^{7x+2} = 4^2$ - Rewrite 16 to have the same base as the other side

$7x + 2 = 2$ - Set exponents equal

$-2 \quad -2$

$\frac{7x}{7} = \frac{0}{7}$ - Solve the equation

$x = 0$

2) $x = 2$

$3^{-x+2} = 1$

need bases the same → $3^{-x+2} = 3^0$

set exponents = → $-x + 2 = 0$

$+x \quad +x$

$2 = x$

3) $x = \frac{1}{2}$

$9^{x+2} = 9^{3x+1}$

$x + 2 = 3x + 1$ - Just set exponents =

$-x \quad -x$

$2 = 2x + 1$

$-1 \quad -1$

$\frac{1}{2} = \frac{2x}{2}$ $\quad x = \frac{1}{2}$

4) $x = -\frac{1}{2} = -0.5$

$5^{6x} = \frac{1}{125}$ ← Fraction means the exponent will be negative

$5^{6x} = 5^{-3}$

$\frac{6x}{6} = \frac{-3}{6}$

$x = -\frac{3}{6} = -\frac{1}{2}$

5) $x = -2/7$

$4^{x-1} = 8^{3x}$

$(2^2)^{x-1} = (2^3)^{3x}$ Need to rewrite both sides as powers of 2

$2^{2x-2} = 2^{9x}$ - Multiply exponents to simplify

$2x - 2 = 9x$ - Set exponents =

$-2x \quad -2x$

$-2 = 7x$

F. Modeling - Word Problems

1) A. Growth - She's getting bigger

B. $h(M) = 5(2)^{\frac{M}{9}}$

$a = 5$ ft. - her starting height
$b = 2$ since she's doubling in size
$\frac{M}{9}$ - you need to divide by 9 since she only doubles every 9 minutes (not every minute)

C. 20 ft

Plug in 18 for M

$h(18) = 5(2)^{\frac{18}{9}} = 5(2)^2 = 20$

2) 15 or 16 bacteria

First, write an equation

$a = 1{,}000$ bacteria to start
$b = 1/2$ since half of the bacteria die off $*1/2$ are left

$y = 1{,}000(\frac{1}{2})^x$

Then, plug in 6 for the 6 hours

$y = 1{,}000(\frac{1}{2})^6 = 15.625$

G. Writing Equations from a Table

1) $y = 20\left(\frac{1}{4}\right)^x$ or $y = 20(0.25)^x$

$a = 20$ since the table has the point $(0, 20)$

$b = \frac{1}{4}$ because I'm dividing by 4 to go from one y-value to the next, which is the same as multiplying by $1/4$

or $\frac{5}{20} = \frac{1}{4}$

2) $y = 10(2)^x$

$a = 10$ since the table has the point $(0, 10)$

$b = 2$ since I multiply by 2 to go from one y-value to the next

or $\frac{20}{10} = 2$

H. Writing Equations Given 2 Points

1) $y = 4 \cdot 6^x$

$(0, 4)$ gives me that $a = 4$

Since the x-values are consecutive (0 then 1), I can just find b by saying $4 \times 6 = 24$

2) $y = 8\left(\frac{1}{2}\right)^x$ or $y = 8(.5)^x$

$(2, 2)$ gives me that $a = 8$

I need to plug in x and y and solve for b because the x's aren't consecutive

$y = a \cdot b^x$

$y = 8 \cdot b^x$ From $(2, 2)$ $x = 2$ $y = 2$

$2 = 8 \cdot b^2$

$\frac{2}{8} \quad \frac{8}{8}$

$\sqrt{\frac{1}{4}} = \sqrt{b^2}$ or $\sqrt{25} = \sqrt{b^2}$ $\quad b = 0.5$

3) $y = \frac{7}{3}(3)^x$

or $y = 2.\overline{3}(3)^x$

I can find b since the x-values are consecutive, but I need to solve for a since they didn't give me a point with $x = 0$

$y = a \cdot b^x$

$y = a \cdot 3^x$ (1, 7) $x=1$ $y=7$

$7 = a \cdot 3^1$

$\frac{7}{3} = \frac{3a}{3}$ $a = \frac{7}{3} = 2.\overline{3}$

x	y	
0	7/3	$\div 3$
1	7	$\times 3 = b$
2	21	

4) $y = 6(1.793)^x$

I know $a = 6$ because of (0, 6), but need to solve for b because the x's aren't consecutive

From (4, 62) I know $x = 4$ $y = 62$

so $y = a \cdot b^x$

$y = 6 \cdot b^4$

$\frac{62}{6} = \frac{6 \cdot b^4}{6}$ Now solve for b

$\sqrt[4]{10.\overline{3}} = \sqrt[4]{b^4}$ on calc:

4, Math, 5: $\sqrt[4]{}$, 10.333, enter

$b \approx 1.793$

I. Even More Modeling

1) $p(x) = 75{,}000\,(0.913)^x$

$a = 75{,}000$ since that's the # of deer at the first census
This is like the point $(0, 75{,}000)$

They say 2 years later there are 62,500 deer. This is like the point $(2, 62{,}500)$. I'll use these #s for x and y.

Plug in a, x, and y then solve for b.

$y = 75{,}000 \cdot b^x$

$\frac{62{,}500}{75{,}000} = \frac{75{,}000}{75{,}000} \cdot b^2$

$\sqrt{.833} = \sqrt{b^2}$

$b \approx .913$

2) $f(n) = 300\,(3)^{\frac{n}{4}}$

$a = 300$ - # bacteria I start with
$b = 3$ - My population is tripling
$\frac{n}{4}$ - divide the exponent by 4 because the tripling only happens every 4 hours

3) $f(n) = 4{,}500\,(0.92)^n$

$a = 4{,}500$ since that's the amount of waste they start with

$b = 100\% - 8\% = 92\% \Rightarrow 0.92$ as a decimal

↑ subtract because they are reducing their waste output = decay

) $v = 35{,}000\,(0.78)^{10}$

$a = \$35{,}000$ - start value of the car

$b = 100\% - 22\% = 78\% \Rightarrow 0.78$ as a decimal

↑ depreciating is decay ⇒ so subtract

The exponent is my # years so I plug in the 10 there.

5) $A = 5126\left(1 + \frac{.205}{12}\right)^{12 \cdot \frac{4}{12}} = \$5485.36$

6) $A = 12,348\left(1 + \frac{.035}{4}\right)^{4 \cdot 1} = \$12785.89$

J. Logarithm Basics

1) $7^2 = 49$ $\quad \log_7 49 = 2$ (7: base, 49: #, 2: exponent)

2) $5^3 = X$ $\quad \log_5 X = 3$ (5: base, X: #, 3: exponent)

3) $10^3 = 1,000$ $\quad \log 1,000 = 3$ (when no # is written, the base is 10)

4) $x^y = a$ $\quad \log_x a = y$ (x: base, y: exponent)

5) $\log_4 64 = 3$ $\quad 4^3 = 64$ (4: base, 3: exponent)

6) $\log_8 1 = x$ $\quad 8^x = 1$ (8: base, x: exponent)

7) $\log y = x$

8) $\log_x h = c$

K Using Logarithms - Rules & Evaluating

1) $5^0 = x + 2$

$x = -1$

$\log_5 (x+2) = 0$ (5: base, 0: exponent)

$\Rightarrow 5^0 = x + 2$

$1 = x + 2$

$-2 \quad -2$

$-1 = x$

2) A. $\log 1.000$

When logs are added, multiply what's inside

$\log 10 \cdot 100 = \log 1,000$

B. $\log 0.1$ or $\log \frac{1}{10}$

When logs are subtracted, divide what's inside

$\log \frac{10}{100} = \log \frac{1}{10}$

C. $\log 10^{10}$

When there is a # in front of the logarithm, it becomes the exponent

3) A. $x = 3$

$\log_2 8 = x$

$2^x = 8$

B. $x = -3$

$\log_3 \frac{1}{27} = x$

$3^x = \frac{1}{27}$

C. 8

$e^{\ln 8}$

the e and ln cancel

4) A. $x = 15,625$

$\log_5 x = 6$

$5^6 = x$

B. $x = 6$

$\log_4(x+6) = \log_4(2x)$

$x + 6 = 2x$

$-x \quad -x$

$6 = x$

C. $c = 2.5$

$\log_2 6 + \log_2 c = \log_2 15$

$\log_2 6c = \log_2 15$

$\frac{6c}{6} = \frac{15}{6}$

$c = 2.5$

L. Solving Using Logs and Exponentials

1) $10^{2x+1} = 4$

$\log 10^{2x+1} = \log 4$

$2x+1 = \log 4$

$x = \frac{\log 4 - 1}{2} \approx -0.199$

2) $\frac{5}{5}\ln(3x) = \frac{25}{5}$

$[\ln(3x)] = [5]$

$e \quad\quad e^5$

$3x = e^5$

$x = e^5/3 \approx 49.471$

3) $\log_3\left(\frac{x+2}{x-4}\right) = 4$

$3^4 = \frac{x+2}{x-4}$

$81 = \frac{x+2}{x-4}$

$81x - 324 = x + 2$

$\frac{80x}{80} = \frac{326}{80}$

$x = 4.075$

4) $5e^x = 2$

$\ln(e^x) = \ln\left(\frac{2}{5}\right)$

$x = \ln\left(\frac{2}{5}\right)$

$x \approx -0.916$

5) $10 + \log_7(x-3) = 12$

$-10 \quad\quad -10$

$\log_7(x-3) = 2$

$7^2 = x - 3$

$49 = x - 3$

$x = 52$

6) $\log_2(x+2) + \log_2(x) = 3$

$\log_2(x^2+2x) = 3$

$2^3 = x^2 + 2x$

$8 = x^2 + 2x$

$x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

$x = -4, 2$

$x = 2$